Erratum to: "A simplified proof of a Liouville theorem for nonnegative solution of a subcritical semilinear heat equations", J Dyn Diff Equat (2009)21:127-132.

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In our previous paper [Nou09], we consider the following semilinear heat equation

$$u_t = \Delta u + |u|^{p-1}u \tag{1}$$

and we give a simple proof of the following Liouville theorem in the nonnegative case:

Theorem 1*(Merle-Zaag) Assume that*

$$p > 1 \text{ and } (N-2)p < N+2. \tag{2}$$

Consider u a solution of (1) defined for all $(x,t) \in \mathbb{R}^N \times (-\infty, T)$. Assume in addition that $|u(x,t)| \leq C(T-t)^{-\frac{1}{p-1}}$, for some constant $C > 0$. Then $u \equiv 0$ or there exists $T_0 \geq T$ such that for all $(x,t) \in \mathbb{R}^N \times (-\infty, T)$, $u(x,t) = \pm\kappa(T_0 - t)^{-\frac{1}{p-1}}$ with $\kappa = (p-1)^{-\frac{1}{p-1}}$.

The simplified proof that we give in [Nou09], cannot be completed without an extra assumption. This is because in line 1 page 131, we assumed that the blow-up solution v had a blow-up point. This is in fact true by Theorem 5.1 in [GK89] if we add the following assumption in line 30 page 128

In this note, we found that in the nonnegative case (treated in [MZ98]) under the additional condition

$$\exists \bar{t} < T \text{ such that } u(\bar{t}) \in H^1(\mathbb{R}^N) \text{ (H).}$$

Of course, with (H), the conclusion of the Liouville theorem we should reach is that the solution u is identically zero. Even though our additional hypothesis reduces the possibilities in the conclusion of the Liouville theorem, we would like to mention that the aim of our paper [Nou09] is to present this more simple proof, which is pedagogically easier then the analysis of [MZ98]. Of course, for unsigned solutions, without the hypothesis (H) we cannot escape the proof given in [MZ00], which heavily relies on the preceding paper [MZ98].

References

[GK89] Y. Giga and R. V. Kohn. Nondegeneracy of blowup for semilinear heat equations. *Comm. Pure Appl. Math.*, 42(6):845–884, 1989.

[MZ98] F. Merle and H. Zaag. Optimal estimates for blowup rate and behavior for nonlinear heat equations. *Comm. Pure Appl. Math.*, 51(2):139–196, 1998.

[MZ00] F. Merle and H. Zaag. A Liouville theorem for vector-valued nonlinear heat equations and applications. *Math. Ann.*, 316(1):103–137, 2000.

[Nou09] N. Nouaili. A simplified proof of a liouville theorem for nonnegative solution of a subcritical semilinear heat equations. *J. Dyn. Diff. Equat*, 21(1):127–132, 2009.

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